APMA 1650 – Homework 5

Due Thursday, July 21, 2016

Homework is due during class or by 3:45 pm in the homework drop box in 182 George St.
Show all of your work used in deriving your solutions.

1. Suppose that the random variables Y_1 and Y_2 have joint density function:

$$f(y_1, y_2) = \begin{cases} y_1 + y_2 & 0 \leq y_1 \leq 1, 0 \leq y_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show this is a valid joint probability density.
   (b) Find the marginal densities for Y_1 and Y_2.
   (c) Find the probability that $Y_1 < 1/2$ and $Y_2 > 1/2$.
   (d) Find the conditional density for Y_1 given $Y_2 = y_2$.
   (e) Find the probability that $Y_1 < 1/2$ given $Y_2 = 1/2$.

2. Suppose that the random variables Y_1 and Y_2 have joint density function:

$$f(y_1, y_2) = \begin{cases} c(1 - y_2) & 0 \leq y_1 \leq y_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the value of c that makes this a valid joint probability density.
   (b) Find the marginal densities for Y_1 and Y_2.
   (c) Find the conditional density for Y_1 given $Y_2 = y_2$.
   (d) Find the expected values $\mathbb{E}(Y_1)$ and $\mathbb{E}(Y_2)$.
   (e) Find the conditional expected value $\mathbb{E}(Y_1|Y_2 = y_2)$.

3. Suppose that the random variables Y_1 and Y_2 have joint density function:

$$f(y_1, y_2) = \begin{cases} e^{-(y_1+y_2)} & y_1 > 0, y_2 > 0 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Find the marginal densities for Y_1 and Y_2. What kind of random variables are Y_1 and Y_2?
   (b) Find the conditional density of Y_1 given that $Y_2 = y_2$ for $y_2 > 0$.
   (c) Are Y_1 and Y_2 independent? Justify your answer.

4. You are again the quality control manager for the Acme Widget Company. You have just launched a new line of MiniWidgets. Your MiniWidgets are produced by a Mini-Widget machine. The MiniWidgets produced by the machine have masses which are normally distributed with a standard deviation of 0.2 grams. The machine can be adjusted so that the MiniWidgets it produces have an average mass of μ grams. What setting for μ should you use so that the masses of the MiniWidgets will exceed 10 grams at most 2% of the time?

5. Let X_1 and X_2 be two independent geometric random variables, both with parameter p. Find a nice, closed-form formula for

$$\mathbb{P}(X_1 = i | X_1 + X_2 = n)$$

Your formula should not involve a sum. Hint: use the definition of conditional probability.