

APMA 1650 – Homework 7

Due Monday, August 1, 2016

Homework is due during class or by 3:45 pm in the homework drop box in 182 George St.
Show all of your work used in deriving your solutions.

1. Let Y_1, Y_2, \dots, Y_n be a random sample from a population with probability density function parameterized by θ given by

$$f_{\theta}(y) = \begin{cases} \theta y^{\theta-1} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$ is the parameter of interest.

- (a) Show that the sample mean \bar{Y} is an unbiased estimator for $\frac{\theta}{\theta+1}$.
 - (b) Show that the sample mean \bar{Y} is a consistent estimator for $\frac{\theta}{\theta+1}$.
2. Let Y_1, Y_2, \dots, Y_n be a random sample from a population with probability density function parameterized by θ given by

$$f_{\theta}(y) = \begin{cases} (\theta + 1)y^{\theta} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > -1$ is the parameter of interest. Find an estimator for θ using the method of moments.

3. Let Y_1, Y_2, \dots, Y_n be a random sample from a population with probability density function parameterized by θ given by

$$f_{\theta}(y) = \begin{cases} (\theta + 1)y^{\theta} & 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > -1$ is the parameter of interest. Find the maximum likelihood estimator (MLE) for θ . Compare this to your answer from the previous problem.

4. Once again, you are the quality control manager for the Acme Widget Company. Your line of MiniWidgets has been so successful that the MiniWidget machines are running nonstop to satisfy the high customer demand. For a properly functioning MiniWidget machine, the probability of producing a defective MiniWidget is 1% (or less). As part of the quality control process, you will take a sample of 100 MiniWidgets from a machine to determine whether it needs repair. To make a statistically-sound decision, you design a hypothesis test to aid you in this process. You desire a level of $\alpha = 0.05$ for your hypothesis test.

- (a) State the alternative hypothesis, null hypothesis, and test statistic for your hypothesis test.

- (b) You sample 100 MiniWidgets from one of your machines and find that 3 of them are defective. At the level of $\alpha = 0.05$, does this machine need to be repaired?
 - (c) What is the p -value for this hypothesis test?
5. A random sample of 500 measurements on the length of stay in hospitals had a sample mean of 5.4 days and a sample standard deviation of 3.1 days. A federal regulatory agency hypothesizes that the average length of stay is greater than 5 days.
- (a) Do the data support this hypothesis with a level of $\alpha = 0.05$?
 - (b) What is the p -value for this hypothesis test?
 - (c) Using the rejection region found in the previous part, calculate β for the specific value of the alternative hypothesis $\mu_a = 5.5$.
 - (d) How large should the sample size be if we require that $\alpha = 0.01$ and $\beta = 0.05$, where we use the specific value of the alternative hypothesis $\mu_a = 5.5$.